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The Minimum Principle and the Perception of Absolute, Common, and Relative Motions

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Wheel-generated motions have served as a touchstone for discussion of the perception of wholes and parts since the beginning of Gestalt psychology. The reason is that perceived common motions of the whole and the perceived relative motions of the parts are not obviously found in the absolute motion paths of points on a rolling wheel. In general, two types of theories have been proposed as to how common and relative motions are derived from absolute motions: one is that the common motions are extracted from the display first, leaving relative motions as the residual; the other is that relative motions are extracted first leaving common motions as the residual. A minimum principle can be used to defend both positions, but application of the principle seems contingent on the particular class of stimuli chosen. We propose a third view. It seems that there are at least two simultaneous processes—one for common motions and one for relative motions—involved in the perception of these and other stimuli and that a minimum principle is involved in both. However, for stimuli in many domains the minimization of relative motion dominates the perception. In general, we propose that any given stimulus can be organized to minimize the complexity of either its common motions or its relative motions; that which component is minimized depends on which of two processes reaches completion first (that for common or that for relative motions); and that the similarity of any two displays depends on whether common or relative motions are minimized.

Philosophers and psychologists have noted parallels between dynamic processes postulated within natural science theories and processes in perception (Köhler, 1969). The analogy proves to be quite rich in several regards (Sober, 1975), particularly with respect to the great value placed on simplicity. Scientific theories strive for simplicity, and given a choice between two theories with equal scope the scientific community seems to choose the simplest; moreover, good perceived patterns tend to be simple (Attneave, 1954; Garner, 1970), and given a choice between two perceiv-

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able patterns of an ambiguous stimulus, the perceptual system seems to choose the simplest (Hochberg & McAlister, 1953; Hochberg & Brooks, 1960). As a legacy from Gestalt theory, the simplicity heuristic in perception is typically called the Minimum Principle. This rule of perceptual organization is one focus of the present paper.

The other focus is motion. In examining effects of motion on figural coherence, many researchers have emphasized the importance of a threefold distinction needed to describe perceived kinematic relations: (a) *absolute motion* of each element in a dynamic display, (b) the *common motion* of the whole configuration relative to the observer, and (c) the *relative motion* of each element to other configural elements. In general, researchers agree on at least two propositions with regard to these motions. First, absolute motions are often not seen as such; most accounts suggest that only relative and common motions are usually perceived when viewing events. Second, the following equation holds for a given element in a dynamic display:

$$\text{common motion} + \text{relative motion} = \text{absolute motion}. \qquad (1)$$

As one can see from this formulation, it is a trivial problem to determine absolute motions given the other two motion components. All that is required is to add common and relative motions together. To the contrary, the problem of extracting common and relative motions from absolute motions is quite deep: there are an indefinite number of common and relative motions that could produce exactly the same absolute motion. Thus, one puzzle for perceptual research is to discover how the perceptual system determines these two motion components.[1]

Not surprisingly, there are differing views on how common and relative motions are perceptually derived from kinematic displays. In particular, the order in which they are extracted has generated debate: are common motions prior, or are relative motions prior? If common motions are prior, then relative motions are the residual derived from subtracting the common from the absolute motions. Likewise, if relative motions are prior, common motions are the derived residual. Our experiments address this issue and provide evidence favoring the priority of relative motions for most stimuli and the priority of common motions for a few others. Employing a minimum principle, we go on to present a hybrid view. First, however, we consider the three types of motion in relation to theoretical positions about their perception.

ABSOLUTE MOTION AND THE CONSTANCY HYPOTHESIS

The absolute motion of an element is its movement without regard to any other element, once an observer-relative frame of reference is

[1] This is a problem analogous to the problem of induction in philosophy (Popper, 1962), and marks a second parallel between perception and theory generation in science.

specified. Thus, absolute motion is simply the trajectory of an element through a set of spatial coordinates over time. The absolute motions of three points on a rolling wheel are shown in Fig. 1a. The absolute motion of perimeter point *A* follows a path called a cycloid, point *B* follows a curve called a prolate cycloid, and point *C* at the center of the wheel follows a linear path. Notice that absolute motions describe the movements of points without regard to the whole of which they are parts.

The first structuralist position in perceptual psychology, the Leipzig view (Boring, 1952), favored the priority of parts over relations applicable to wholes. This view, embodying what Koffka (1935) called the *constancy hypothesis,* assumed that "things look as they do because the proximal stimuli are what they are" (p. 80), and that "the result of a local stimulation is constant . . . that all locally stimulated excitations run their course without regard to other excitations" (pp. 96–97) (see also Hochberg, 1957, p. 73). Thus, it would seem that to defend the Leipzig view, one must argue for the priority of absolute motions over relative and common motions since the latter two movements are dependent on topographic and kinematic relations between parts.

Gestalt psychologists found the constancy hypothesis an easy foil where motion perception was concerned. Rubin (1927) and Duncker (1938), through examinations of simple and complex rotational motions, formulated an oft-repeated Gestalt dictum: One does not perceive the motion of a uniform whole by perceiving individually the movements of its various parts. Rubin noted that when a rolling wheel is viewed its center is seen to have a translational motion, whereas all other points on the wheel are typically seen as revolving in circles about it. This perception for the three points on a rolling wheel depicted in Fig. 1a is illustrated in Fig. 1b.

Duncker placed a single light on the perimeter of an unseen wheel in an otherwise dark room and found that the absolute, cycloidal motion of this

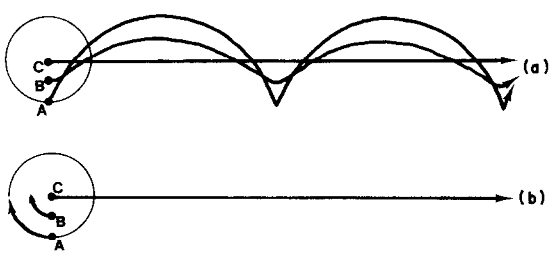


FIG. 1. (a) The absolute motion of three points mounted on an unseen rolling wheel. Point *A* describes a cycloid, point *B* a prolate cycloid, and point *C* a line (or completely degenerate cycloid). (b) The typical perception of this configuration: points *A* and *B* have relative circular motion about point *C*, and the figure as a whole has common motion of linear translation across the field of view.

point was observed. However, when a second light was added to the wheel at its center a quite different set of motions was perceived. Observers reported three configurations: two lights mounted on a wheel rolling across a flat terrain, the perimeter point looping about the center which again moved linearly, or the two points moving together like a "tumbling stick" with lights attached to each end. In this manner the absolute motion of the center point was perceived in the first two cases but not in the third, and the absolute motion of the perimeter point was not observed. Results such as these are thought to demonstrate the failure of the constancy hypothesis and the unlikelihood that absolute motions are necessarily perceived prior to movements dependent on wholes. We shall return to this stimulus event and its perception many times, and we shall also suggest that absolute motions do play a role in the perception of these displays.

COMMON MOTION AND THE MINIMUM PRINCIPLE

Common motion is the perceived movement of a whole object relative to an observer. Every element in the whole shares this motion. However, for no element in the whole is it necessary that common and absolute motions correspond. Discussion of three examples will be helpful at this point. First, consider a block moved linearly and normal to the direction of an observer's gaze. The common motion seen is translation and the absolute motion of every elment in the block is fully described by this movement. Thus, given Eq. (1), this is a case where relative motion is null and common and absolute motions are identical. Second, consider a whole wheel as it is observed rolling along a path normal to the observer's gaze. Again, the common motion is translation; however, in this example the center of the wheel is the only element for which the common and absolute motions correspond. Every other point on the wheel has a rotational component as well as the shared common translation. Third, consider an array of points of light moving as if attached to the joints of a walking person. The common motion is the walker's locomotion–horizontal translation with the addition of a slight lunging and undulating motion. The only point within the walker that equates this motion with its absolute motion is a point within the torso of the individual, and no light need mark this point for the common motion to be perceived (Cutting, Proffitt, & Kozlowski, 1978). Every point-light in the array shares in this common motion as well as the concatenation of various pendular, twisting, and bobbing motions.

Hochberg (1957) looked to common motion as a likely candidate for a perceptual minimum principle. Earlier, Köhler (1920/1938) had suggested that physical Gestalten such as soap bubbles and raindrops tended toward states requiring "minimum energy" for their maintenance. However, he could provide few psychological examples of this tendency toward Prägnanz. Koffka (1935) summarized Köhler's proposal and further urged psychologists to seek psychological phenomena that conformed to a minimum principle. At the 1954 Cornell Symposium on Perception there was greatest agreement on the importance of the minimum principle, and common motion was cited as exemplifying this tendency toward simplicity. Hochberg (1957, p. 82) wrote, "In Johansson's [1950] motion studies, those components in a complex moving stimulus which are common to all members of a group are 'partialled out' and form a single framework in relation to which the residual motions appear. Such unification achieves an 'informational' economy since, for any given stimulus, the percept entailing the least number of changes is obtained." In order for the perceptual system to employ a minimum principle in selecting common motions the information specifying these motions must be partialled out first from the event, prior to all others, causing relative motions to be specified by the residual.

The idea that the common vector is the first motion to be extracted from kinematic displays has been attributed, perhaps incorrectly, to Johansson[2]. Regardless of the origin of this view, however, Johansson (1973) chose to investigate an historically important stimulus, the perception of which can indeed support the priority of common motions: Duncker's (1929/1938) phenomenon of an unseen wheel rolling with one light at its center and one on its perimeter. According to Johansson, "When, in the motion of a set of proximal elements, equal simultaneous motion vectors can be mathematically abstracted (according to some simple rules), these components are perceptually isolated and perceived as one unitary motion" (1973, p. 205). Thus, the common motion of translation is extracted leaving the rotational component of the perimeter light as residual. Selecting linear translation as the common motion minimizes this motion component but not relative motion vectors. We will return to discuss this stimulus and the minimum principle, but first we must present an opposing view.

[2] In several places Johansson can be read to support the priority of common vectors (Johansson, 1950, p. 135; 1973, p. 207; 1976, p. 386), whereas in others he can be read to support the priority of element-relative vectors (Johansson, 1950, p. 96; 1958, p. 362; Johansson, von Hofsten, & Jansson, 1980, p. 33). In fact, however, when he has written about priority of common vectors he has described a mathematical process that may (or may not) mimic the perceptual one, and when he has written about priority of relative vectors, he has described attentional salience. In fairness to Johansson, it is probably best to say that he has made no explicit statement on perceptual process and the order of extraction of relative and common vectors. Despite this, Gogel (1974, p. 426), Hochberg (1957, p. 79), Hochberg and Fallon (1976, p. 1081), Kalveram and Ritter (1979, p. 398), Proffitt, Cutting, and Stier (1979), Proffitt and Cutting (1979), and Rock (1975, p. 216n) all attribute the common-vector-prior view to Johansson.

RELATIVE MOTION AND THE MINIMUM PRINCIPLE

The relative motion of an element is its movement with respect to other configural elements. Consider again the three examples discussed previously. In the first, where the block is translating normal to the line of sight of an observer, there are no observed relative motions since the absolute motion of every element is equivalent. However, should the block be viewed to move over some distance as it translates, left to right, then a relative motion of rotation will be observed as the forward and trailing surfaces of the block are concealed and revealed by occlusion and disocclusion, respectively. In the second example, the rolling wheel has two perceived components of motion: a translational component that is its common motion relative to the observer, and a rotational component that is defined by the relative motion of every point to the center of the motion generating wheel. In the third, the array of lights is moving as if attached to the joints of a walker, and a nesting of relative motions is observed (Cutting & Proffitt, 1981). Considering only one leg, the ankle is seen moving with a half-pendulum motion relative to the knee. In like manner, the knee swings through a pendulum arc relative to the hip; and finally the hip is observed to swivel about a point within the torso. This scheme is generally the same when the other leg and the upper body are added. Relative and common motions are thus distinguished by the former specifying movements of elements relative to points within the configuration. The latter, on the other hand, specify equivalent motions of all configural elements relative to an observer.

Wallach (1965/1976) proposed a scheme consistent with the idea that relative motions are extracted from events prior to common motions. In discussing the perception of rolling motions, Wallach chose to investigate a stimulus with two lights placed diametrically opposite one another on the rim of an unseen rolling wheel. He and his observers reported that rotation and linear translation were perceived. Although Wallach asserted the priority of relative motions, for this particular stimulus the perception of these same two components of motion would occur even if common motions were extracted first. Thus, this stimulus is not a crucial test of the priority of relative or common motions since both orderings yield equivalent descriptions. However, as with Duncker's (1929/1938) wheel event, this latter stimulus will be returned to frequently in our subsequent discussions.

Rock (1975), Gogel (1974), and Börjesson and von Hofsten (1975) also argued for the priority of relative motions. After analyzing some of Johansson's demonstrations, Rock (p. 215) concluded that relative motions were perceptually more dominant than observer-relative common motions. Gogel presented to observers a display with three moving dots: one dot moving horizontally and the other two moving vertically, 180° out of phase, and at opposite ends of the horizontal excursion of the first dot. He found that the horizontal dot was seen to manifest, alternately, the relative motions induced by each vertically moving dot, depending on which was nearer. (However, see Restle, 1979, pp. 11–12, for other perceived interpretations of this event.) Börjesson and von Hofsten presented to observers arrays of 3 to 4 point-lights undergoing various motion patterns in order to examine the perception of translation and rotation in depth. Their work was directed by a vector analysis model that placed a priority on reducing the sum of relative motions to zero. Thus, relative motions were made primary in determination and their selection was specified by a minimum principle.

Our own work on perceiving wheel-generated motions is also consistent with the relative-vector-first view as well as the selection of relative motions via a minimum principle as suggested by Börjesson and von Hofsten (Proffitt et al., 1979; Proffitt & Cutting, 1979, 1980a, 1980b). The results of these studies may best be summarized through a discussion of Fig. 2. The absolute motion of two points that are 90° apart on the rim of an unseen rolling wheel is shown in Fig. 2a. Depending on whether a minimum principle operates on common or relative motions first, two different sets of motion patterns would be observed. If common motions are extracted

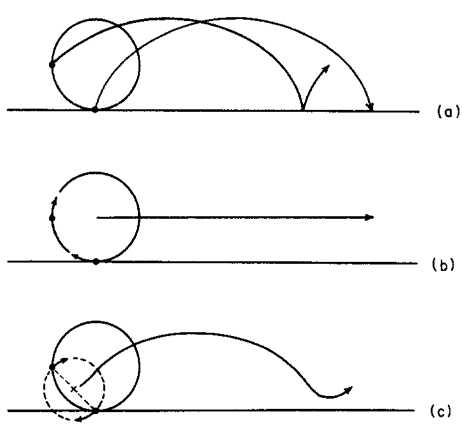


FIG. 2. (a) The absolute motion paths of two lights mounted 90° apart on the perimeter of an unseen rolling wheel. These yield cycloids 90° out of phase. (b, c) Two possible perceptions of this stimulus, depending on whether common motion is extracted prior to relative motion or vice versa. (b) Where common motion is extracted first, one would see linear translation across the field of two lights mounted 90° apart. (c) Where relative motion is extracted first, the two lights are seen as if mounted on a smaller wheel, 180° out of phase, and that wheel traversing a prolate cycloidal path. Our previous research suggests that only the version in (c) is seen by naive observers, and it supports the notion that relative motions are extracted prior to common motions, and that a minimum principle is operating on the organization of relative motions (see also Börjesson & von Hofsten, 1975).

first, then the motions drawn in Fig. 2b should be observed. Linear translation is the minimum possible common motion of both points,[3] and its extraction leaves the rotation of each point about the center of the wheel as the residual relative motion. If, on the other hand, relative motions are prior, then the motion paths drawn in Fig. 2c should be seen. The relative motion of each point with respect to the other is a rotation about the point midway between them. Extracting these relative motions first leaves the motion of this configural midpoint as the residual common motion. The results of our research suggest that the latter set of motions are perceived, and suggest further that, contrary to Hochberg (1957), the minimum principle might be applied to relative, not common, motions.

Let us consider this last point in more detail. For the perception of rotational motions, at least, it seems a reasonable conjecture that a minimal principle operates on relative motions. The keystone of our previous work is the idea that the relative motions are perceived to occur about the centroid of the configuration. If these motions are extracted first, then selection of the centroid as center for rotation minimizes two interrelated aspects of the dynamic stimulus: one of movement and one of spatial extent.

First, rotation about the centroid causes the sum of *momentary relative motions to equal zero*. Consider the four-light stimulus shown in Fig. 3a. Since this configuration spins about its centroid, the sum of the four vectors, one per light, always equals zero at any time. This is not so for the rotation of the same configuration shown in Fig. 3b. Here, the center of rotation is displaced away from the centroid, and it can be seen that the four vectors for this stimulus would not sum to zero, but would sum to a rotational vector of relatively small radius. (The radius, in particular, would be the distance of this rotational center from the centroid of the configuration.) This fact accrues importance because it is general: regardless of the number of points or their density in a rigid configuration, rotation about the centroid causes all relative motions to sum to zero. Rotation about any other point fails to do this. In this manner, centroid rotation realizes one type of minimum principle for perception.

Second, rotation about the centroid causes the sum of *the squared lengths of moment arms to all points to be minimal*. Although this fact necessarily follows from the first, it is worth separate consideration. The same stimulus is shown in Fig. 3c as in Fig. 3a, but here the radii for each rotational vector is indicated. For practical purposes these are normalized to unity, and sum of squares is 4.0. The same stimulus is shown in Fig. 3d

[3] Linear translation should be considered the minimum common motion because it is the minimal trajectory, or path, between two end points. Moreover, it is the simplest trajectory to specify by equation, requiring the fewest parameters.

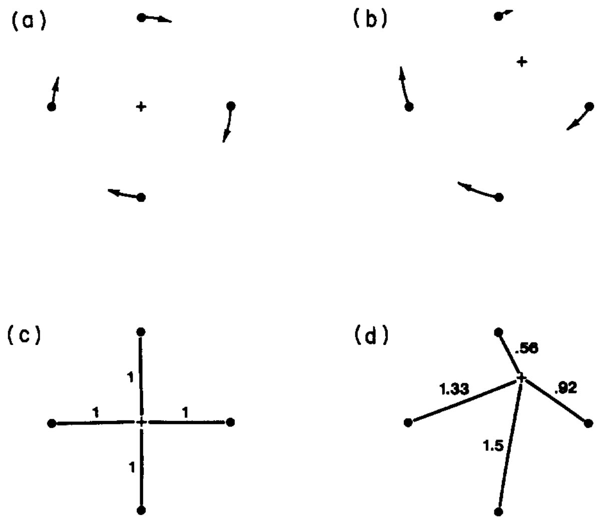


FIG. 3. (a) The relative vector paths for four points of light rotating (a) about their centroid and (b) about another point displaced from their centroid. Only in (a) do momentary vectors always sum to zero, thus realizing one manifestation of a minimum principle. (c, d) The relative lengths of moment arms to these four points in both versions. Only in (c) is the sum of the squared lengths of moment arms minimized, thus realizing a second type of minimum principle.

as in Fig. 3c but with a reference point displaced from the centroid. Notice that the sum of the squared lengths of these moment arms is 5.18. This, of course, means that the total area delimited by the rotation of the four lights will be substantially greater. Moreover, the farther the center of rotation is from the centroid the larger the sum of the squared lengths of the moment arms and the greater the summed areas. Again, this is true regardless of the number or density of points in the configuration. Thus, although not independent of the first fact, centroid rotation also realizes this second type of minimum principle for perception. We have discussed both of these aspects of minimization found for centroid relative rotations because we suspect that perceptual processes could be sensitive to only one type of information and not the other; moreover, we have at present no evidence reflecting on relative sensitivity to either aspect of minimization.

Overview. In summary, then, we proffer three ideas. First, complex motion perception generally consists of perceiving common and relative motions which, when vectorially combined, sum to the absolute motions in the display. Second, a minimum principle is an important heuristic for motion perception. Third, a minimum principle may be operative in each of two processes, one involving common motions and one relative motions. In the next section we develop the last of these ideas.

TWO CONCURRENT PROCESSES FOR MOTION PERCEPTION

Having described the three types of motion, let us now turn to the issue of order of extraction of relative and common motions. Before beginning, however, it must be emphasized that we are speaking of a difference in solution time of two simultaneous processes each operating in accordance with a minimum principle. We do not assume that the first p msec of any perceptual process is devoted exclusively to the extraction of one type of motion, and the next q msec devoted exclusively to the other. Instead, our notion may be best viewed as entailing two concurrent perceptual processes, one dealing with common motion and one dealing with relative motion. From Eq. (1), one can see that common and relative motions co-constrain one another. That is, by vector addition they must add up to the same absolute motion. Therefore, if one process reaches solution first, the incomplete solution by the other process is negated because that motion is determined residually. In this manner, we suggest that if relative motions are first to be minimized by the perceptual system, common motions fall out as residuals. If, on the other hand, the perceptual system first minimizes the common motion, relative motions fall out as residuals. One motion component is product, the other byproduct. In most cases it can be determined which is which by the observer's report on what is perceived since only one of the motion components will be minimized.

Two Stimuli Generating Wheellike Motion

Two stimuli are of particular interest to our study of wheel-generated motions. They have been discussed previously and are shown in Fig. 4. Both consist of two lights mounted on an unseen wheel. The two lights of stimulus (a) are mounted on the perimeter 180° apart (see Wallach, 1965/1976), and stimulus (b) has one light on the perimeter and one at the center (see Duncker, 1929/1938; Johansson, 1973, 1974a, 1974b; Börjesson & von Hofsten, 1975). Recall that it was through consideration of these different stimulus events that different accounts of the priority of motions have been (or can be) proposed—(a) for the priority of relative motions and (b) for the priority of common motions. We wish to look more carefully at the perception of these stimuli in relation to one another and to the perception of other configurations.

Consider how these two stimuli are often perceived. Stimulus (a) appears to have only one stable set of perceived motions: lights mounted on a rolling wheel with rotation and linear translation. Nothing in the literature and nothing in our experience suggests any other easily perceived

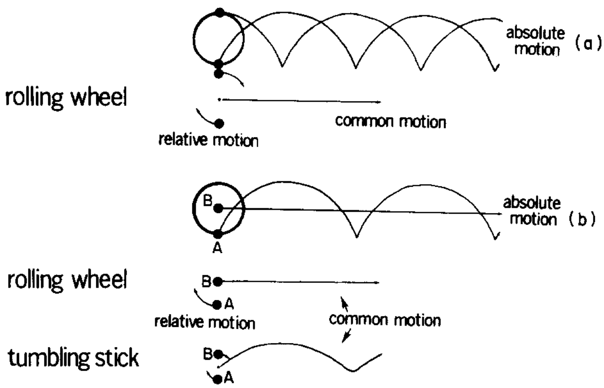


FIG. 4. Two stimuli used as prototypes to describe wheel-generated motion. (a) A two-light stimulus with lights mounted 180° opposite from one another on an unseen wheel rim. The absolute motion paths are two cycloids, 180° out of phase. The relative motion paths, on the other hand, are circular and 180° out of phase around their midpoint. Common motion is the path of this midpoint, which is linear. (b) A two-light stimulus with one light on the perimeter and one at the center. Absolute motion paths are a straight line and a cycloid. The relative and common motion paths, however, depend on the particular version of the object seen, either as a rolling wheel or as a tumbling stick. In the rolling wheel version, relative motion occurs only for light A, rotating about light B. Common motion occurs for both and is linear. In the tumbling stick version, on the other hand, both lights have relative motions, rotating 180° out of phase around their midpoint, and they both have common motion, describing a prolate cycloid.

motion paths (although there exists a set comprised of an indefinite number of relative and common motions that are compatible with the absolute motions of these two cycloidal arcs). Stimulus (b), on the other hand, is perceptually multistable. Duncker (1929/1938), for example, noted that it could be perceived as having either wheellike motions or as moving like a "tumbling stick" (see also Börjesson & von Hofsten, 1975). The tumbling stick motions are shown at the bottom of Fig. 4, and the rolling wheel version is directly above. Notice that although the absolute motions are the same, the relative and common motions are different for each. In the rolling wheel version, light A has a relative motion of rotation around light B, whereas light B has no relative motion. Both share common motions of linear translation. Here, common motion is minimized. In the tumbling stick version, on the other hand, the perceived movement of two lights minimizes relative motions and moment arms. They rotate, 180° out of phase, around a common unseen center and they share common motion along a prolate cycloidal path. Johansson's (1973) account suggests that the perception of the linearly rolling wheel is preferred for stimulus (b).

We have conducted several studies using these stimuli and many others

(Proffitt et al., 1979; Proffitt & Cutting, 1979, 1980a). In general we have presented the moving point-light configurations to individuals and asked them to judge "how wheellike" the movements of each appeared. Our results were highly regular and suggest that viewers perform some geometric—or even calculuslike (Proffitt & Cutting, 1980b)—operations on the moving configuration, determine its centroid, and then reckon the movement of this unseen central point against a fixed background, such as the edge of the video monitor. The less the centroid moves up or down in its otherwise lateral excursion across the screen, the more wheellike the configuration appears; the more it hops up and down, the less wheellike it appears. We developed an index to measure this vertical excursion called D_m/r, or the *d*istance from the configural *m*idpoint (or centroid) to the center of the motion generating wheel, divided by the *r*adius of the rotating configuration as measured from the center of the wheel to its outermost point of light. The logic of determining this index for a two-light configuration is shown in Fig. 5. In more than a dozen experiments we found viewers' judgments of the individual stimuli highly correlated with stimulus indices. Average correlations were .90 or better.

More pertinent to this discussion, we found that stimulus (a) of Fig. 4 was perceived as more wheellike in its movements than stimulus (b) (Proffitt et al., 1979, Experiment 1). This result fits our general scheme because stimulus (a) has a D_m/r index of .00 (the midpoint of the system of

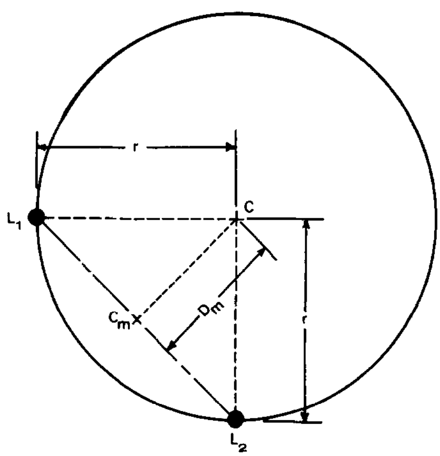


FIG. 5. The method for determining our distance metric D_m/r is to determine first the centroid of the light configuration. Since this is a two-light configuration, the centroid is the midpoint between the two lights L_1 and L_2. This point is marked C_m, corresponding to the center of moment of the system. D_m is the distance from the generating center of the wheel (the axle) to the midpoint, and r is the distance of the lights from the axle. The D_m/r index for this stimuli is .71.

lights lies directly at the center of the generating wheel) and stimulus (b) has an idex of .50 (its midpoint lies half the distance to the outermost light, the one on the perimeter of the wheel). However, stimulus (b) was perceived as more wheellike than our metric predicted, perhaps because of its multistable nature. Only the tumbling stick version of stimulus (b) would be perceived if our D_m/r metric captured all possible perceptions.

One methodological problem is this set of studies concerns its dependent measure—rating-scale judgments along the dimension of "wheellikeness." This objection divides two ways. First, the use of rating scales is susceptible to criterion shifts, which could substantially affect the results. This criticism suggested to us a different methodology, one used by Proffitt and Cutting (1980a). There, among other tasks, we presented all possible pairs of stimuli to viewers who made forced choice judgments concerning either the wheel likeness or hopping motion of the stimuli. Results were entirely congruent with those of our previous studies. Thus, the use of rating scales per se seems not to have created artifactual results. The second problem with the dependent measure used in these studies, however, applies equally well to the latter study and the earlier ones: the use of an exceedingly small number of dimensions—"wheellikeness" and "hopping motion." Whereas the unusual nature of these dimensions does not cause us worry, the fact that viewers were forced to use these and no others does raise concern. In other words, it seems feasible that our stimuli could be judged regularly along a dimension called wheellikeness or hopping motion, and yet be judged equally well along some different, unspecified dimension that would yield discrepant results. Thus, the demand character of the task might easily have biased the results.

Our current study, then, has a twofold purpose. First and foremost, we wish to compare the perceptual roles and processes for common and relative movements, drawing on stimuli often used as prototypes for wheel-generated motion. Second, we wish to use an unbiased response measure that is unlikely to presuppose our results. To accomplish this latter goal we asked observers to rate the general similarity of movement of various pairs of stimuli, without suggesting to them the dimensions on which they should base their judgments. These data were then compiled into matrix form and scaled multidimensionally. The interpretation of the dimensionality of the results allows us to recover the stimulus dimensions that observers appeared to use.

GENERAL METHOD

Thirty individuals from the Wesleyan University community participated in three experiments, 10 in each. Few had any experience with motion perception experiments, and most were paid for their services.

Their general task was to judge the similarity, on a 7-point scale, of sequentially presented pairs of stimuli displayed on a video monitor.

Stimuli were generated on a DEC-12 computer and displayed on a VR11 oscilloscope. They consisted of one- and two-light configurations mounted as if on an unseen wheel rolling in the picture plane across the oscilloscope face from left to right. Parallel projection was used. The first member of each pair rolled across the top half of the display face, followed immediately by the second which rolled across the bottom half. All wheels for all stimuli had the same radius; thus, all lights generated cycloidal (if mounted on the rim), prolate cycloidal (if mounted in the interior of the wheel, but not at the center), or linear (if mounted at the center) absolute motion paths. All stimuli made three revolutions in transit across the monitor. Each revolution took 833 msec, yielding a stimulus presentation of 2.5 sec and a trial duration of 5 sec per pair. There was a 4-sec pause between trials to allow participants to write their response. Stimulus sequences were video recorded and played back to viewers on a 28-cm video monitor. Viewers generally participated in groups of two, watching two different random orders of 64 stimulus pairs each. In each experiment these 64 consisted of all possible pairs of 8 stimuli. Participants were allowed to view several practice trials before the experimental session began in order to familiarize them with the stimuli and task. Visual angle of each point of light was 15 min of arc; visual angle of the entire stimulus trajectory was 6 degrees.

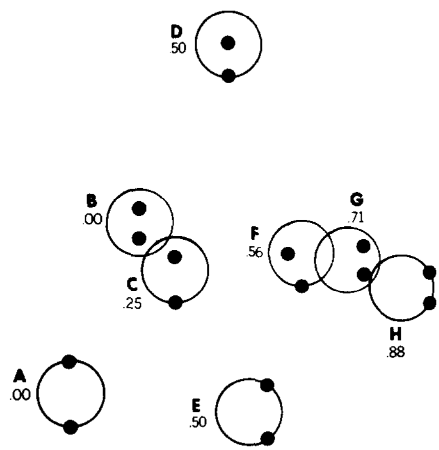


FIG. 6. The two-dimensional solution for the eight stimuli, *A*–*H*, used in Experiment 1; stress = .08. The center of each stimulus is located at the two coordinates specifying the placement of that stimulus in the solution. D_m/r indices for each stimulus are given in the figure; see the text for their explanation. Stimuli *A* and *D* are identical to stimuli (a) and (b) in Fig. 4.

Judgments for all viewers in each experiment were averaged and placed into an 8 by 8 matrix. This matrix was then averaged across the major diagonal, the diagonal removed, and entered into the nonmetric multidimensional scaling program KYST (see Kruskal & Wish, 1978). No special weights were given to any stimulus judgments.

EXPERIMENT 1: COMMON AND RELATIVE MOTIONS

Stimuli

The eight stimuli used are shown in Fig. 6. Stimuli *A* and *B* have D_m/r indices of .00 and have two lights mounted 180° opposite from one another. They differ only in that in stimulus *A* the lights are mounted as if on the perimeter of the wheel and in stimulus *B* they are mounted half the length of the radius out from the center. Stimulus *C* has a D_m/r index of .25, also with lights 180° opposite, but with one on the perimeter and one halfway out from the center. Stimuli *D* and *E* have indices of .50, with stimulus *D* having one light at the center and one at the perimeter, and stimulus *E* with two lights on the perimeter 120° apart. Stimulus *F* has an index of .56, with lights 90° apart, one on the perimeter and one a half-radius from the center. Stimulus *G* has an index of .71, with two lights 90° apart both at .71 radii from the center. Finally, stimulus *H* has an index of .88 with two perimeter lights 60° apart. Notice that stimuli *A* and *D* are identical to stimuli (a) and (b), respectively, in Fig. 4.

Results and Discussion

The two-dimensional solution for the judgments to these stimuli is shown in Fig. 6. Before discussion of its dimensions, however, it should be noted that the stress (or badness-of-fit) for this solution is quite acceptable (.08), that the stress for the one-dimensional solution is appreciably higher (.26), and that for the three-dimensional solution is somewhat lower (.04) but without an easily interpretable third dimension[4].

Displayed in this manner the horizontal dimension is readily seen as reflecting D_m/r indices. Indeed, there is a perfect rank–order correlation between location of a stimulus on this axis and its D_m/r index. We consider this a happy result that strongly corroborates our earlier work. In essence, once relative motions are extracted from the dynamic displays, differences in common motion are arrayed along a continuum seen as the horizontal dimension of Fig.6.

The vertical dimension is less clearly interpretable. As we see it, it may be construed in either of two ways. Interpretation I suggests that this dimension may capture the degree to which any light in the two-light system is mounted toward the center of the generating wheel. That is, stimuli toward the bottom of the figure have both lights mounted on the

[4] All stress values are computed by Stress Formula 1 (e.g., Kruskal & Wish, 1978, p. 26). Also, another reason that three-dimensional solutions are not considered here is that, in general, it is recommended that the number of objects scaled not be less than four times the number of dimensions considered (Kruskal & Wish, 1978, p. 52). With eight stimuli in each experiment, we are thus confined to two dimensions.

perimeter; stimuli in the middle generally have at least one light mounted halfway from the center (stimulus G has two lights at .71 radii from the center, and stimulus H is a clear exception with both lights on the perimeter); and stimulus D at the top of the display has one light at the center. On the other hand, interpretation II suggests that dimensions are less important than local clusters, and that Fig. 6 displays only the relative dissociation of stimulus D from all other stimuli.

Support for interpretation I comes from the scaling solution derived from the data matrix when all stimulus D comparisons are removed. Indeed, this solution is very similar to that shown in Fig. 6 with stimulus D absent. Such a configuration suggests that stimuli A and E lie on a different line than stimuli B, C, and $F-H$. The two parallel lines denote either the proximity of the two lights, or the extent to which one light is mounted interior to the perimeter. The latter interpretation is, of course, the same as interpretation I for the vertical dimension of full solution. In other words, the relative arrangements shown in Fig. 6 do not appear to derive from spatial deformations due to the inclusion of stimulus D. Support for interpretation II, on the other hand, comes from cluster analysis of these results. Both maximum and minimum method solutions show two strong clusters: one composed of stimuli $A-C$ with low D_m/r indices, and the other of stimuli $E-H$ with higher D_m/r indices. Stimulus D is a definite outlier to both these clusters.

Regardless of which interpretation is favored, both converge on the same fact: stimulus D, with one light on the perimeter and one at the center, is not a central member of the stimulus set. In fact, stimulus D is perhaps the least representative of these stimuli in that it occupies the lowest density region of the solution (Krumhansl, 1978). It should also be noted that stimuli A and D were judged to be the least similar of all stimuli in this set. Remember, these two stimuli are the ones used to support opposite theories about the extraction of motion components.

It struck us that since differences in common motions are arrayed in the horizontal dimension, something about relative motion might be captured in the vertical.

EXPERIMENT 2: COMMON, RELATIVE, AND ABSOLUTE MOTIONS

The special status of stimulus D in the results of Experiment 1 piqued our curiosity as to how it is perceived. Two possible explanations occurred to us. First, it may be the multistability of stimulus D, an attribute unique in this stimulus set, that caused its outlying position in the scaling solution. Second, it may be the dissociation of relative motions in one of the perceived configurations of the stimulus (the rolling wheel version favored by the common-vector-first theory) that caused it to be an outlier. That is, relative motion is perceived to be present only in light A in Fig. 4

which is seen revolving around the center (light B). The common motion is shared equally by both lights, and consists of linear translation. In this manner, light A contains both motion components but light B only one. No other stimulus would be perceived this way, as we will discuss later.

The current study was designed to corroborate the second possibility. In particular, stimulus D and four other stimuli from Experiment 1 were compared with one another and with three new stimuli, each consisting of a single light. The rationale is that a one-light stimulus has no relative motion. That is, it consists only of movement of the whole. Remember, Eq. (1) dictates that without relative motion, common motion equals absolute motion. Indeed, reports by Duncker (1929/1938), Koffka (1935), and Johansson (1973) suggest that this is how single lights are perceived, and our experience reaffirms it. If the central light of stimulus D is perceived to be similar to a one-light stimulus, this evidence would be consistent with the idea that stimulus D overtly contains the perceived common motion in one of its lights, and that the rolling wheel version, not the tumbling stick, is predominant. Moreover, in the dimensional solution the relative proximities of the three one-light stimuli, stimulus D, and the other four two-light stimuli should inform us more on the representativeness of stimulus D among two-light stimuli.

Stimuli

Stimuli A, B, D, E, and G were identical to those used in Experiment 1. Again, they have D_m/r indices of .00, .00, .50, .50, and .71, respectively. Added to these five were three one-light stimuli: stimulus X had its light mounted at the center of the wheel and its excursion across the monitor screen simply traced a straight line with uniform motion. Stimulus Y had its light mounted a half-radius from the center and it traced a prolate cycloid, and stimulus Z had its light on the perimeter and traced a true cycloid.

The D_m/r index measures the distance of the centroid of light cluster (which for these last three stimuli is the center of each single light) from the movement generating center of the system and then divides that measure by the distance to the furthermost light. Thus, for stimuli X–Z, and indeed for all one-light stimuli, the D_m/r index is 1.00. In this manner, the index is not appropriate for distinguishing configurations of less than two lights.[5] Nevertheless, in terms of the absolute paths of the lights in the one- and two-light stimuli, interesting comparisons can be made. In particular, the absolute motion of the one-light of stimulus Z is identical to the absolute motions of each of the two lights in stimuli A and E, and to the one perimeter light of stimulus D. The extent to which this stimulus lies in proximity to these two-light stimuli will reflect the extent to which judgments of similarity are based on absolute motion of the light of stimulus Z and those of stimuli A, D, and E. In addition, the absolute motion of stimulus Y is identical to the movements of each of the lights in stimulus B, and quite similar to these of stimulus G. Moreover, the absolute movement of stimulus Y is identical to the movement of the centroid of stimuli D and E. Likewise, stimulus X is identical in movement to the central light in stimulus D, and to the common motion of the

[5] With this particular formulation of the D_m/r index, which we presented first in Proffitt and Cutting (1979), some one-light stimuli in a previous experiment (Proffitt et al., 1979, Experiment 2) are given incorrect indices.

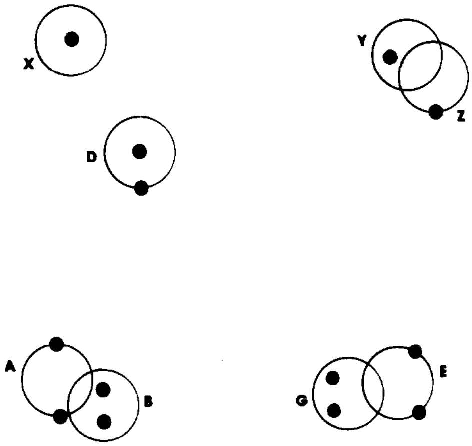


FIG. 7. The two-dimensional solution for the eight stimuli—A, B, D, E, G, and X–Z—used in Experiment 2; stress = .09. Stimuli A, B, D, E, and G are the same as those used in Experiment 1.

centroids of stimuli A and B as well. As we will see, however, only one of these anticipated relations holds in any striking way—the similarity of stimuli D and X.

Results and Discussion

The two-dimensional solution for these stimuli is shown in Fig. 7. Stress here is acceptable (.09), whereas that for the one-dimensional solution is not (.31), and that for the three-dimensional solution is quite good (.03), but without a readily interpretable third dimension.

This solution essentially divides the stimuli into four nearly equidistant clusters, so let us consider cluster analysis first. Stimuli A and B group together, as we suggest they should since both have D_m/r indices of .00. Likewise, stimuli E and G form a second group, and since their D_m/r indices are relatively high—.50 and .71, respectively—this too is sensible.[6] Stimuli Y and Z form a sensible third cluster, since they have prolate cycloidal and cycloidal absolute motions, respectively, of single lights. This leaves stimuli X and D as a fourth, more loosely allied group.

A dimensional, rather than cluster, analysis of the stimuli yields as interesting an interpretation. The horizontal dimension as shown can be

[6] The relative placement of these four stimuli is quite different from that in Fig. 6, and deserves some comment. Here, they have formed two tightly knit groups with stimuli E and G reversed in ordinal position. Yet given that the inventory of stimuli has changed across the two studies, the difference between them should not be particularly bothersome. Indeed, stimuli F and H of Experiment 1 have D_m/r indices closer to that of stimulus G than does stimulus E, and they appear to have drawn stimuli G and E apart in Fig. 5.

construed as the distance of the centroid of the system from the center of the generating wheel. (Remember: for one-light configurations this is not D_m/r, even though it is strongly correlated with D_m/r for configurations of more lights.) Stimulus G is the only item spoiling an otherwise perfect rank–order correlation along this axis. Again, such an interpretation supports our previous work.

The vertical dimension can easily be interpreted as the number of lights in the system, with the bottom of the figure representing two-light systems and the top representing one-light systems. In this manner stimulus D can be said to be perceived more like one-light stimuli than like other two-light systems. It seems doubtful, however, that this can be the entire story.

The peculiarity of stimulus D both here and in Experiment 1 invites further speculation of both the dimensionality of this solution, and on the underlying principles of how this stimulus is perceived. Symmetry might appear to be promising for the interpretation of the solution, but we believe it is not. In one sense, the four stimuli at the bottom of Fig. 7 are all symmetric. If one drew a line between the two lights of each configuration, then bisected that line, the bisector would pass through the center of the generating wheel. Not so for stimulus D. But we reject this interpretation of the results for two reasons. First, perpendicular bisectors for stimuli C and F in Experiment 1 do not pass through the center of the wheel, and yet they were perceived to be quite like the others of that stimulus set. We suggest that these stimuli, were they included in this experiment, would lie at the bottom of Fig. 7, with stimuli A, B, E, and G. Second, the symmetry of the four stimuli at the bottom of the figure does not suggest any positive reasons why stimulus D should lie near the one-light stimuli rather than far away from them. One other scheme, more speculative at this point, may capture the dimensionality here. It focuses on one aspect of relative motions.

Shared relative motions. Stimuli A, B, E, and G share certain attributes. One we think pertinent is that the two lights in each system share equally in both common motion and in the absolute value of their relative motions. Consider stimulus G. As most observers see it, this systems consists of two lights held in rigid relation to one another in the picture plane, turning circles around their centroid and hopping (or bouncing) across the monitor screen in an arclike fashion. As we conceive it, the rotational motions about the centroid are the relative motions. These motions are circular, have equal radii, and are exactly 180° out of phase. This minimizes relative motions (they sum to zero) and squared lengths of moment arms. The hopping motion, or vertical excursion due to the nonzero character of the D_m/r index, is one aspect of the common motion of the whole. Together with horizontal translation, the two motions describe a prolate cycloid and like the absolute value of the relative motions, this motion is shared equally by both members of the system. This general relative motion–common motion description works for all four of these stimuli. The analysis is the same for stimulus *E*, and it differs for stimuli *A* and *B* only in that their common motions are linear rather than cycloidal. Thus, those stimuli at the bottom of Fig. 7 consist of light patterns that share equally in relative motion (circular rotations differing 180° in phase) and in common motion (observer–relative movement of linear or cycloidal form). This description would also apply to the tumbling stick version of stimulus *D*, were it perceived. That is, both lights would be in rotation about their midpoint and both would traverse a cycloidal path, as shown in Fig. 4. However, results suggest that the tumbling stick version is not seen, or at least not preferred.

The rolling wheel version of stimulus *D* does not fit the foregoing description. As previously mentioned, only the perimeter light (light *A* of stimulus (b) in Fig. 4) has relative motion—it rotates about the center of the generating wheel. The other light, of course, is *at* the center, and thus its relative motion is null. It has only the common motion of linear translation across the screen. That this version seems to be preferred over the tumbling stick is a failure of our general scheme for wheel-generated motions (but see Proffitt et al., 1979, Experiment 4). Yet this failure is an interesting one. Among the eight different two-light systems that are presented here, and among the many others that we have presented elsewhere, this stimulus is the *only* systematic exception to our D_m/r scheme, and it is the only one for which the two lights do not share equally in the absolute value of their relative motions.

This account allows a new interpretation for the vertical dimension of Fig. 7. Those stimuli at the bottom of the figure have no lights without relative motions, whereas those at the top have at least one light that has no relative motion. In the latter category, for stimulus *D* it is the center light, and for the other stimuli, of course, it is the only light that these configurations contain. Expressed in a more positive manner, then, those stimuli at the top of the figure have lights that reveal the common motion directly in an absolute motion, whereas those at the bottom have no lights that do this. Common motions are directly visible for stimuli *D* and *X*–*Z* where, for at least one element, its absolute motion *is* common motion; the absolute motion is not equivalent to the common motion for any element in stimuli *A*, *B*, *E*, and *G*.

We acknowledge this account as speculative. Moreover, it could be rejected simply by suggesting that the data presented in Fig. 7 are not really amenable to dimensional analysis, only cluster analysis. Therefore, to render a dimensional view more plausible we conducted a third experiment.

EXPERIMENT 3: INTERACTION OF ABSOLUTE AND COMMON MOTIONS

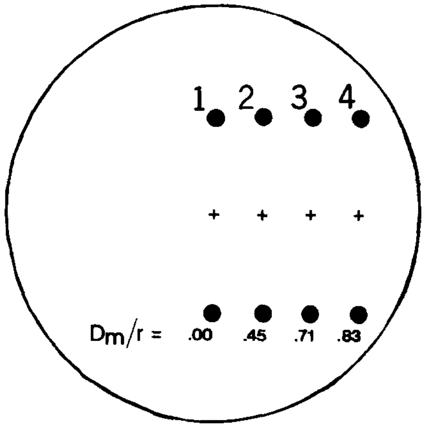
Our concern in this third study stems from one aspect of the dimensional interpretation of the results of Experiment 2. Scrutiny of Fig. 7 will be necessary to the following discussion. Consider second-order similarities. One can see that stimuli *X* and *D* are roughly as similar to stimuli *A* and *B* as are stimuli *Y* and *Z* to stimuli *E* and *G*. More particularly, following our argument outlined above, stimuli *X* and *D* have common motions directly visible in the movement of one light, and the trajectory of that common motion is linear. Stimuli *A* and *B* have motions from which linear common motion may be derived, once relative motions are extracted from absolute motions. Again, the common motion of each of these four stimuli is linear translation.

The other four stimuli are congruent in this same manner, but with different common motions. Stimuli *Y* and *Z* directly reveal common motions that are cycloidal in some form, arcing across the video monitor. Stimuli *E* and *G*, on the other hand, have absolute motions from which a prolate cycloidal common motion may be derived once relative motions are subtracted out. Thus, the common motion of each of these four latter stimuli is cycloidal or prolate cycloidal.

What concerns us is this second-order similarity. It seems unlikely that the similarity between direct vs derived *linear* common motion (stimuli *X*, *D* vs stimuli *A*, *B*) would bear the same proximity relations as direct vs derived *cycloidal* common motion (stimuli *Y*, *Z* vs stimuli *E*, *G*). Instead, we would predict the latter to be considerably more similar. The rationale for this expectation is less opaque than might first appear, and stems from three facts. First, the absolute motions of all lights in all stimuli in this paper are either cycloidal, prolate cycloidal, or linear. Second, the common motions are also either cycloidal, prolate cycloidal, or linear. Third, relative motions are always circular. Given these facts, we would expect that when common motion is held constant, similarity judgments should be positively related to the degree to which common motion is matched by the absolute motions of the lights in the system. More concretely, when common motion is more cycloidal, one- and two-light configurations ought to be similar; when common motion is more linear, one- and two-light stimuli ought to be less similar. In other words, we expect absolute motion to play some role in perception.

Stimuli

The eight stimuli consisted of two subsets: four were two-light systems and four one-light systems, as shown in Figs. 8a and b, respectively. Stimulus 1 consisted of two lights 180° apart mounted a half-radius from the center; stimulus 2 of lights 126° apart at .56 radii from the center; stimulus 3 of lights 90° apart at .71 radii; and stimulus 4 of lights 67° apart at .90



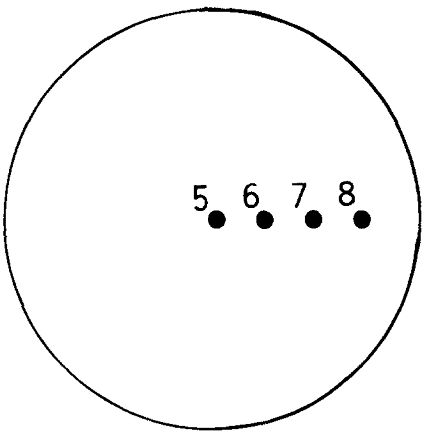


FIG. 8. The eight stimuli used in Experiment 3: (a) four two-light stimuli (1–4) and (b) four one-light stimuli (5–8). Pluses denote the midpoint, or centroid, for each of the two-light stimuli. These midpoints match the locales of the single lights of stimuli 5–8. Stimuli 1, 3, 5, and 7 are the same as stimuli *B, G, X,* and *Y* of Experiment 2.

radii. The D_m/r indices are .00, .45, .71, and .83, respectively. Stimuli 1 and 3 of this set are identical to stimuli *B* and *G* from Experiments 1 and 2.

Stimuli 5–8 consisted of single lights mounted .00, .25, .50, and .75 radii from the center, respectively. Thus stimulus 5 and 7 are identical to stimuli *X* and *Y* of Experiment 2. The four locations of lights in stimuli 5–8 correspond to the midpoints of the systems of lights for stimuli 1–4. In other words, the absolute movements of stimuli 5–8 correspond to the common movements of stimuli 1–4. Yet absolute movements for the two groups are uncorrelated. In fact, the absolute movement of stimulus 7 is exactly that of each of the lights of stimulus 1. Relative movements, of course, occur only for stimuli 1–4, where they are identical for all stimuli. That is, each light pair circles around the centroid 180° out of phase from one another at a distance exactly equal to one half their separation. Since all pairs of lights are equidistant, all relative motions should be the same.

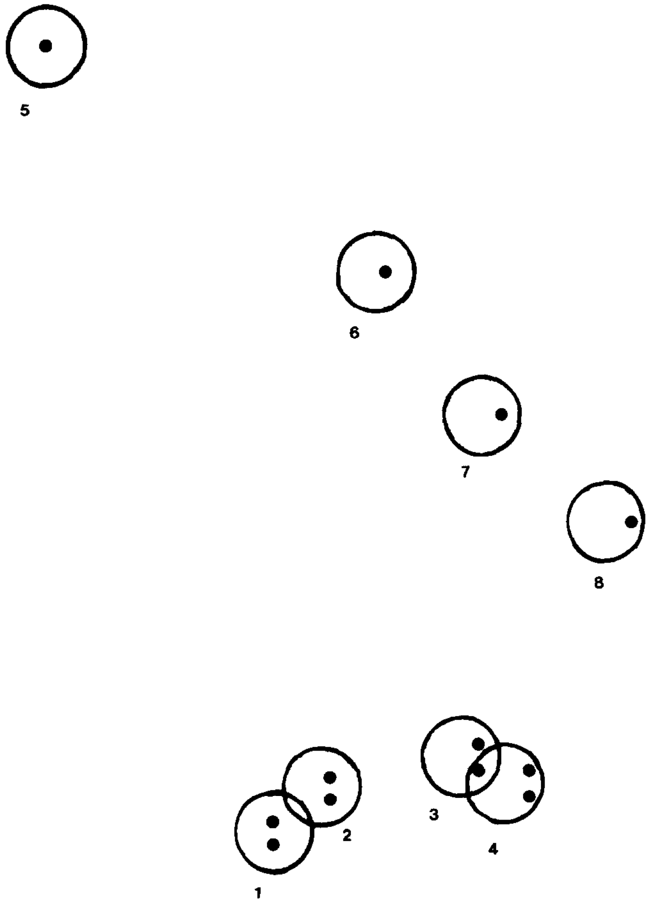


FIG. 9. The two-dimensional solution for the eight stimuli (1–8) of Experiment 3; stress = .01.

Results and Discussion

Figure 9 shows the two-dimensional solution for these eight stimuli. Stress here is extremely low (.01), but this is not unduly surprising given the manner in which the stimuli were selected. The stress for the one-dimensional solution is relatively satisfactory (.12), and that for three-dimensions no better than that for two (.01).

The horizontal dimension is readily interpretable as the distance of the centroid of the configuration from the generating center of the wheel. For stimuli 1–4 this is D_m/r arrayed left-to-right with increasing indices. Stimuli 5–8 follow suit, except that they are spaced apart to a greater degree. Again, such an arrangement supports our previous work.

The vertical dimension, on the other hand, is much more intriguing. A first approximation suggests that it simply separates two-light stimuli from one-light stimuli, just as it did generally in Experiment 2 (Fig. 7). However, since the two subsets of stimuli each array themselves almost linearly in the solution, it is noteworthy that their linear arrangements are by no means parallel. Instead, if regression lines through the two subsets were drawn and extended, they would intersect at an angle of nearly 60°. Such a result suggests a powerful interaction between the perception of a moving whole with elements in relative motion (stimuli 1–4), and the perception of a moving whole without the relative motions (stimuli 5–8).[7]

Absolute, common, and relative motions. Pair comparison of stimuli across the two groups reveal that stimuli 4 and 8 are more similar than stimuli 3 and 7, which are in turn more similar than 2 and 6. Stimuli 1 and 5 are clearly the least similar of all four pairs. These comparisons reveal further that stimulus 5 is roughly equally dissimilar to all four of the two-light stimuli, suggesting that it is perceived quite differently. Analyses of the absolute motions of all these stimuli begin to suggest the reason for this two-dimensional arrangement.

Absolute motions of the one-light stimuli grade from linear (stimulus 5) to varying degrees of prolate cycloids, with vertical oscillation depths of .25, .50, and .75 radii of the motion-generating wheel (stimulus 6–8, respectively). Absolute motions of each of the lights in the two-light stimuli are all prolate cycloidal with depths of .50, .56, .71, and .90 radii (stimulus 1–4, respectively). In this manner, only the one-light stimuli 7 and 8 have absolute motions within the range of these for the two-light stimuli. Thus, that stimuli 7 and 8 are closest to stimuli 1–4 seems reasonable. By this analysis, then, it would appear that absolute motions may be important in judging moving stimuli, since they provide a good account of the results here. However, this factor must have had a weak and secondary influence, since the absolute motions of lights in stimuli 1 and 7 are identical and yet stimulus 1 was judged to be least similar to stimulus 7 of all two-light stimuli.

In summary, then, the interpretation of Fig. 9 appears to require the analysis of three factors. First, as in the previous experiments, the horizontal axis seems attributable to the differential common motions of the whole. Second, the relative clustering of stimuli 1–4 seems attributable to

[7] At this point, a comparison of Figs. 7 and 9 is instructive. As mentioned earlier, stimuli *B*, *G*, *X*, and *Y* of Fig. 7 are exactly the same stimuli as stimuli 1, 3, 5, and 7, respectively, of Fig. 9. Yet Fig. 9 shows this interaction and Fig. 7 does not. We suggest that this may be due to the inclusion of stimulus *D* in the set of stimuli used in Experiment 2, where it may have deformed the spatial representation of a set of stimuli. The possible deformations in Experiment 2 due to stimulus *D* may be another example of its unusual nature and general nonrepresentativeness.

their sharing exactly the same relative motions, two lights circling 180° out of phase around their midpoint. Of course, no relative motion occurs for Stimuli 5–8, and perhaps this is why they are relatively dispersed and removed from the first cluster. Third, the vertical axis seems attributable to the relative similarity—when common motions are held constant—of the absolute motions of the paris of stimuli 1–5, 2–6, 3–7, and 4–8.

DISCUSSION

Minimum Principles for Relative and for Common Motions

The results of these three experiments strongly support our previous work. The horizontal dimension in each scaling solution is attributable to our D_m/r index, or to some straightforward correlate. The consistency in these results suggests that observers are attentive to the degree to which the centroid of the system oscillates vertically in its otherwise horizontal excursion. This vertical, or hopping, motion is one aspect of the common motion of the system. We suggest that for most stimuli, relative motions are minimized by the perceptual system, often resulting in residual common motions that are far from minimal.

Another interesting finding, however, is the nature of a dimension orthogonal to D_m/r. The vertical dimension of the solutions to all three scaling studies is interpretable in the following manner: stimuli are arrayed according to the degree to which common motion matches the absolute motion of one of the lights. In other words, whereas the horizontal dimension reflects D_m/r (the similarity among common motion once relative motion are extracted), the vertical dimension reflects the similarity of common motion to absolute motion. It is along this second dimension that radical differences can be seen between two stimuli used in support of different theories of motion perception. In an account proposing the priority of relative vectors (Wallach, 1965/1976) a stimulus was chosen—stimulus *A* in Experiments 1 and 2, and stimulus (a) in Fig. 4—in which absolute motion and common motion are maximally different. In an account often thought to propose the priority of common vectors (Johansson, 1973; but see footnote 2) a stimulus was chosen—stimulus *D* in Experiments 1 and 2 and stimulus (b) in Fig. 4—in which absolute and common motions are maximally similar. These facts are revealing when one reassesses the two theories.

The relative-vector-first theory proposes that relative motion precedes common motion in the extraction of information from absolute movements. How is this done for stimulus *A?* The most parsimonious description of their motion relative to one another is that they are 180° out of phase in revolution about a point midway between them. This description is parsimonious because, as previously outlined, it minimizes both the sum of the momentary relative motions and the sum of the squared lengths of moment arms to both lights. Once these relative motions are determined, the residual is linear progression across the face of the video monitor.

The common-vector-first theory, on the other hand, emphasizes that common motions precede relative motions. How might this be done for stimulus *D?* If the center light is taken as the reference point, the peripheral light can be seen to revolve around it. Since the center light contains the minimum common motion of the whole, and no relative motion, the common motion is necessarily prior; it is *identical* to the absolute motion of that light. The relative motion of the perimeter light is determined only after the common motion is subtracted out of its absolute vector.

Thus, the relative-vector account seems to be appropriate for stimulus *A*, the common-vector account for stimulus *D*. What is most interesting, however, is that the accounts are asymmetrically exclusive. That is, the relative-vector account will not work for stimulus *D* because it would predict that it should appear as a tumbling stick. On the other hand, the common-vector account *will* work for stimulus *A* if a minimum principle is invoked to determine the common linear motion before the relative motions. Were these the only two stimuli available, the case might easily be won by the common-vector account as the theory with broader scope. However, in conjunction with other stimuli, the force of data favors the relative-vector theory, where all stimuli but one (stimulus *D*) are accounted for. The common-vector theory, on the other hand, accounts for only those stimuli that are perceived to have perfectly linear, horizontal translation (stimuli *A*, *B*, and *D*).

A quandary exists, then, if one is to account for the perception of all stimuli. An either–or approach to the minimization of common and relative vectors cannot suffice. One way out of this dilemma is to propose that both processes go on concurrently. In this vein the following scheme emerges: the perceptual system simultaneously tries to apply a minimum principle to both common and relative motions. It reaches solution first for common motions when the common motion for the whole and the absolute motion for one element are identical and minimal—as is true for stimulus *D* perceived as a rolling wheel. Failing in this solution, the perceptual system reaches solution first for relative motions when all momentary relative motions sum to zero and when squared lengths of all moment arms from the perceived center of rotation are minimal—as seems to be true for all other two-light stimuli presented here and in our other work (Proffitt et al., 1979; Proffitt & Cutting, 1979, 1980a). This latter scheme works for wheel-generated motions with three lights or more (Proffitt et al., 1979; Proffitt & Cutting, 1979), and even for nonrectilinearly bounded shapes mounted on rolling wheels (Proffitt & Cutting, 1980b).

The exceptional stimulus and two predictions. Two factors make stimulus *D* exceptional. First, it is multistable: in particular, it can be perceived as a rolling wheel or a tumbling stick. Second, the preferred percept appears to be the rolling wheel, one that cannot be accounted for through minimization of relative motions. Both these factors prove interesting for making predictions and setting limiting conditions on motion perception.

Consider first multistability. Our general view is that common and relative motions are complementary. That is, they sum to the absolute motion and this summative quality causes them to co-constrain one another. If one accepts our notion that simultaneous processes try to minimize both common and relative motions, and that the process which reaches its solution first dictates the percept, then both the rolling wheel and the tumbling stick version of stimulus *D* can be accounted for. Can we, by similar means, account for other bistable dynamic displays? We would predict that one perception may be accounted for on the basis of minimizing relative motions and the other by minimizing common motions.

Four corroborating examples may be found in the work of Johansson (1950, Experiments 19–21 and 24), also discussed by Restle (1979).[8] In Experiment 21, for example, two lights are seen to move in linear harmonic motion, one in a vertical path and one in a horizontal path 90° out of phase with the first. The extent of the two paths form a plus sign. Two percepts are generally seen: one is of a pivot light moving horizontally (or vertically) with the other in rotation about it; the other is of two lights mounted 180° opposite one another on the rim of a wheel that is rolling within a hoop of twice the diameter (see Rubin, 1927). In the first case, the common motion is minimized—where the common motion is simply a harmonically repeated linear path equivalent to the absolute motion of one of the lights. The residual relative motion is a circular path seen in the other light with a radius equalling the distance between the two lights. In

[8] Restle (1979) has attempted to formalize a minimum principle for many of Johansson's (1950) demonstrations and for some first created in his own laboratory. Adapting Leeuwenberg's (1971) coding theory to movement, he suggested that perceived configurations of moving clusters of lights involve the specification of fewer parameters of motion than do all component lights considered independently. That is, the more such parameters as phase, plane, tilt, wavelength, and amplitude are shared among the individual moving lights, the more coherent the whole should appear. Yet as Cutting (1981a) has suggested, this approach, despite its elegance, has difficulties. In particular, to code movement (or shape, or any other stimulus property) one must assume a particular coding scheme over all possible alternative schemes imaginable. Restle (1979), for example, adopted a scheme of coding movement in terms of circular movement parameters, certainly a logical choice, but this leaves him with no easy way to code nonrepeating movements or movements unrelated to circles and ellipses, such as the quasi-pendular motions found in gait.

the second case, the relative motions are minimized (they sum to zero) and the squared lengths of the moment arms are minimized (half the distance between the two lights squared, then multiplied by 2). As residual the common motion is a circular path, and is necessarily more complex than a linear path because more parameters are needed to specify it. Johansson's (1950) Experiment 24 is like the previous one, but with a third light added 45° out of phase with the other two and traversing a harmonic linear path at 45° to the other two. Again, two percepts are generally seen: one is of a pivot light moving linearly with the other two rotating about it, and the other is of a three-light wheel rolling within a larger hoop, just as before. Again, common motion is minimized in the first percept and relative motion in the second. Johansson's (1950) Experiments 19 and 20 can also be analyzed in this manner. It is interesting that Restle (1979) codes the two possible percepts of each of these four configurations as having the same prominence value—indicating that neither should be preferred over the other.[9] By extension we would expect an elaboration of Restle's system to code the rolling wheel and tumbling stick versions of stimulus *D* to have the same prominence value.

This leads us to the second aspect of stimulus *D* that proves interesting: the fact that the rolling wheel is preferred over the tumbling stick. Duncker (1929/1938) found that when viewers had no particular fixation point they seemed to perceive only the tumbling stick; however, when they fixed on one of the lights the observers often reported the rolling wheel. Citing Duncker's results, Börjesson and von Hofsten (1975) suggested that a fixation point provides a strong reference frame that can override a minimum principle applied to relative motions when the display is very simple, as in stimulus *D* and in Johansson's (1950) Experiments 21 and 24. Without such a strong reference frame, they suggest, this type of effect would not occur. We wish to push this idea of reference frame still further. Our experiments were not done in dark rooms; instead, observers watched the movement of lights on a video monitor with sides of the display face fully visible. It would seem that these edges would provide a reference frame much more stable than simple eye fixation. Given this stable frame, it is likely that the observer would quickly notice the linear path of the center light of stimulus *D*. With this considered it is somewhat less surprising that the rolling wheel version is apparently so strongly preferred by our observers. A second set of predictions, then, is that when a strong external reference frame is available to the observer who is

[9] Some of Restle's (1979) analyses of Johansson's (1950) multistable demonstrations are not interpretable as minimizations of common versus relative motions, but are merely different versions of minimized common motions. Restle's (1979) Figs. 8B, 8C, and 9C are examples.

presented with a multistable stimulus, the observer will, for this particular stimulus, see more readily the version that entailed minimization of common motion. Conversely, when these cues are not present, the version that entailed minimization of relative motion is likely to be perceived more readily.

We know of little work that is directly relevant to this issue. Restle (1979) reports that the two configurations perceived in each of the four Johansson (1950, Experiments 19–21 and 24) demonstrations mentioned earlier are equally obtainable, with no preference of the minimized common motion percept over the minimized relative motion percept. Johansson (1950), however, reports for his Experiment 19 a percept that is consistent with our relative-motion-first view. However, Johansson has often used a large collimating lens placed in front of the oscilloscope face that not only obscures frame cues but minimizes depth cues as well (e.g., Johansson, 1974b). We would suspect that, under these conditions, percepts with relative motions minimized would predominate. Indeed, using this lens, Börjesson and von Hofsten (1975) report data entirely consistent with the relative motion view. Regardless, we believe this second prediction to be far less important than the first since situations without external reference frames are extremely rare in the natural environment, and hardly frequent even in the laboratory.

We have presented evidence in support of the view that the perceptual system tries to minimize both common and relative motions, and that the relative facility with which this can be done for each motion component dictates the percept in a multistable event. An adjunct to this explanation is that the first motion component to be minimized is extracted from the event and the second motion component is the residual. Thus, for all stimuli consisting of point-lights moving as if attached to a rolling wheel, with the exception of stimulus *D*, relative motions, consisting of rotations about the centroid of the configuration, are minimized first. The motion of the centroid is residual and specifies the common motion of the configural whole. There exists, however, an alternative explanation for our results that requires neither a minimum principle nor the priority in extraction of relative motion.

An Alternative to the Minimum Principle

The centroid relative motions seen in almost all wheel-generated events could be derived by a three-step process: first, the perceptual system initially determines the configural centroid; second, the motion of this point is extracted from the event as the common motion; and third, the relative motions are specified by the residual. The cardinal point of this alternative explanation is the priority of centroid determination. If the centroid of a configuration can be determined from static spatial information then common motion could be extracted first, although by this account relative motions about the centroid could likewise be prior in extraction. That is, if we assume that the perceptual system first derives the centroid using static configural information, then the order of motion information extraciton becomes indeterminate. This proposal, moreover, seems not to require a minimum principle. We do not favor this explanation because it lacks generality in three respects.

The first and most telling objection is that it requires that the perceived center of the event be derived from static information. There are, however, numerous events in which the perceived center is indeterminate without motion information. Such indeterminacy is found in those events with unbounded arrays. The flow of information produced by locomotion in a textured surround is seen to have centers fore and aft that are, of course, indeterminate in static topology. Similarly, motion information is required to determine the rotational center of the revolving night sky. Another instance of the indeterminacy of perceived centers from static information involves events that are produced by nonrigid transformations. A cardioidal transformation applied to the profile of a face is perceived to alter the age of the person depicted (Pittenger and Shaw, 1975); however, the relative changes in the outline are seen as occurring about a center of moment other than the configural centroid of any of its static arrays (Cutting, 1978b). A similar problem occurs in perceiving the center of stimuli with nestings of component structures such as human walker. The perceived center of moment does not correspond to the center of gravity within the walker nor any other point derivable through analysis of a static frame within the step cycle. Each of these events will be discussed in more detail after we have completed our critique of this alternative.

Second, this explanation can be rephrased in terms of a minimum principle, but it fails to permit the perceptual system to make use of relevant motion information, and it requires more processing steps than does our proposal. Analytic derivations of the centroid from spatial information minimize two parameters of the configuration that are corollaries of the minimization of relative rotational motions. Physiological process models of centroid determination employ minimum principles that, through eye movements, reduce the relative distributions of brightness on opposite sides of the fovea (Pitts & McCulloch, 1947; Bruell & Albee, 1955). Pitts and McCulloch, in particular, suggest that the superior colliculus performs a double integration on the distribution of brightness and moves the eyes so that this first moment of the area approaches zero at the fovea. This process causes all momentary relative motions to approach zero as well. Also, the second moment of the area, a measure of compactness (Zusne, 1970), is minimized when the centroid is taken as the point of reference. Moreover, spatial extent of relative motions is, thereby, also minimized. Thus, models operating on either spatial or motion information may employ minimum principles the products of which are perfectly correlated. If motion information is not used in centroid determination then three steps in processing are required: determining the centroid, extracting one of the motion components relative to this point, and deriving the second component as a residual. If motion information is used, and we know from studying the perception of such events as the unbounded arrays discussed above that it can be used, then not only can centroid determination be based on more sources of information, but also only two steps in processing are required: extracting relative motion about the centroid using spatial and kinematic information and deriving the residual common motion. Thus, our proposal allows the perceptual system to make use of all the information to which we know it is sensitive. The alternative explanation does not. Moreover, our proposal requires fewer steps in processing.

Finally, the alternative explanation cannot account for those events in which minimization of common motion is achieved for stimulus *D*. Our approach which allows a minimum principle to operate on both relative and common motion simultaneously affords an explanation that further suggests the use of motion information in the determination of perceived centers of moment.

On the Generality and Importance of Minimizing Relative Motion

We suggest that the perceptual system attempts to minimize both common and relative motion. Nevertheless, of the two processes—one minimizing common motion and the other minimizing relative motion—we suspect the latter is more frequently achieved in everyday perception. We can best document this by citing four very different types of events in which relative motion are first to be minimized and one in which minimization of common motion determines the percept (see also Cutting & Proffitt, 1981; Cutting, 1981b).

The first event type consists of harmonically generated motions, like those presented here. We suggest that, for the most part, the perception of wheel-generated motion discussed by Duncker (1929/1938), Wallach (1965/1976), Johansson (1973, 1974b), Börjesson and von Hofsten (1975), and us can be accounted for by postulating perceptual processes that minimize relative motion. There is, of course, the exception of stimulus *D* but there are few others. In addition, the perception of circular motions in the demonstrations of Rubin (1927), Johansson (1950), and Restle (1979), particularly when the number of lights is greater than three, can best be accounted for by the same principle.

A second event type consists of the proximal stimulation in flowfields generated as one moves through the environment, particularly at high speed (Gibson, Olum, & Rosenblatt, 1955). Within the flowfield there is a fixed point around which everything radiates in exponential fashion. This point, if one is looking ahead, is the point *toward* which one is moving. In a complementary fashion, when one is looking behind, it is the point *from* which one is moving. Relative motions, the radial flow of all elements in the field, are minimal with respect to only these two points. That is, if one were to express the flow pattern in a series of equations with respect to all possible points on the proximal image, the equations would be simplest for these two points, fore and aft. The importance of the location of these points, and hence the importance of minimizing relative motions, is that they tell the perceiver from where and to where she or he is headed. Thus, observer–relative ego motion is derived from a prior minimization process on relative motions.

The third event type consists of profiles of an aging face. Pittenger and Shaw (1975) demonstrated that the aging of a human face can be mimicked by a cardioidal transformation of its profile. This transformation occurs about a point, and through simultaneous manipulation of the location of this point and the extent of the transformation, Cutting (1978b) demonstrated that viewers could determine varying degrees of "goodness" in profiles as reflecting the aging process. The best location for this point can be inferred from successive displays through minimization of the relative changes in the profile. The importance of the proper location of this point, and hence the importance of minimizing the relative changes in the faces, is twofold: first it is very near a point around which growth transformations of the skull actually occur, and second a perceiver could use it in the recognition of people over long spans of their life.

The fourth event type perceived via a minimum principle applied to relative motions is walking. Johansson (1973) demonstrated that when one mounts lights on the major joints of a person and has that person walk laterally across an unlit stage, an observer has no difficulty in perceiving the presence of a human walker. In fact, the human form can be accurately determined within 100 msec (Johansson, 1976). It is the figural coherence disclosed in relative motions, bending arms and bending legs with respect to the torso, that indicates that the moving array of lights is a human being. The perceived center within the torso is derived first by minimizing the motions of the shoulders and hips along the stress lines of the twisting torso (Cutting et al., 1978). The component structures of the upper and lower body are perceived through a minimization of common motions in a manner quite similar to the perception of stimulus D. Following the extraction of shoulder motion, this point becomes a static pivot for perceiving the pendular movement of the elbow. In like manner the elbow then becomes the static center of moment for perceiving the pendular motions of the wrist. In perceiving the nested component structures of the upper and lower body, each step of information extraction causes one point to become static and the null common motion within each subsystem is achieved relative to this point, as was found for stimulus *D*. The common motion of the whole body is seen in the center of moment within the torso. Might not the minimization of common motion be responsible for the determination of this deepest perceptual center? We think not since the only condition for which we can suggest how common motions are minimized occurs when the common motion is identical to the absolute motion of one light. Yet this does not occur for the walkers of Johansson (1973) or of Cutting et al. (1978). Another method for determining common motion without reference to relative motion might be to add the vectors of all motion elements at each instant. It seems unlikely that this would work, however, since number (Kozlowski & Cutting, 1977) and location (Cutting, 1981a) of lights on a walker have remarkably little effect on the percept. Changing these would have systematic effects on the summed vector of all the lights in the display, yet the perceived common motion remains the same.

One event that we have discussed elsewhere (Cutting & Proffitt, 1981) reflects in its perception the function of a minimum principle applied to common motion. This event consists of the slow rotation of the night sky. Migratory songbirds can use the kinematic information in this event to guide their migratory flight in both Fall and Spring (Emlen, 1975). The rotation of the night sky is apparently seen by these birds as occurring about the celestial North and South Poles. That these birds perceive and use this information is corroborated by an experiment conducted by Emlen. When yearling birds are placed at the appropriate time in the Spring within a planetarium that rotates the night sky naturally, they orient toward the polar North Star. However, matched birds placed in the planetarium but observing rotation of the night sky about another star found near the equator, orient toward it as if it were the pole star. The importance of the location of the unmoving central star is that it gives the birds an unmoving reference point that can be invaluable for long night flights over water or dark terrain. The night sky, having a relatively uniform distribution of stars and no configural edges (the horizon is an occluding edge, not a configural edge), presents an event in which all possible sets of relative motions are equivalent in summed magnitudes. That is, relative motions are no more minimized by taking the pole star as center than they would be if any other star were selected as the center of rotation. However, rotation selected as occurring about any other point would cause the whole night sky to manifest the common motion of that reference point. Thus, the night sky presents an event in which the minimization of common motion determines the perception. The pole star equates minimal common motion for the whole night sky with its absolute motion. Minimization of relative motion cannot reach a unique solution since the night sky is unbounded.

In summary, we think that minimum principles operate simultaneously on both common and relative motions and that the first process to reach solution determines both motion components, one directly and the other as a residual. For most rotational events investigated, it is the relative component that directly reflects the minimization process. Although our proposal is based on examination of far too few events, we suspect that this priority of relative motions will prove generally to be in evidence in future investigations on perceiving kinematic relations.

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